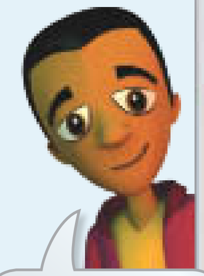
8-1 The Pythagorean Theorem and Its Converse

Common Core State Standards

G-SRT.C.8 Use . . . the Pythagorean Theorem to solve right triangles in applied problems. **Also G-SRT.B.4**

MP 1, MP 3, MP 4, MP 8

Objective To use the Pythagorean Theorem and its converse



Can you use the results of this activity to make a conjecture about triangles?

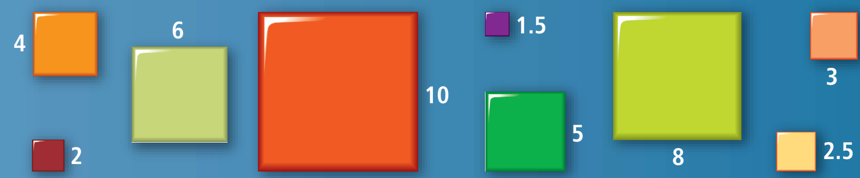
MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

The squares below fit into groups of three to satisfy the following equation.

area of square 1 + area of square 2 = area of square 3

Using each square only once, write an equation for each group. What is the relationship between the three sets of numbers? Explain.



The equations in the Solve It demonstrate an important relationship in right triangles called the Pythagorean Theorem. This theorem is named for Pythagoras, a Greek mathematician who lived in the 500s B.C. We now know that the Babylonians, Egyptians, and Chinese were aware of this relationship before its discovery by Pythagoras. There are many proofs of the Pythagorean Theorem. You will see one proof in this lesson and others later in the book.

Lesson Vocabulary
- Pythagorean triple

Essential Understanding If you know the lengths of any two sides of a right triangle, you can find the length of the third side by using the Pythagorean Theorem.

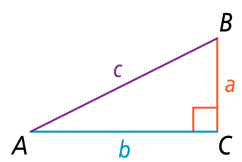
take note — Theorem 8-1 Pythagorean Theorem

Theorem

If a triangle is a right triangle, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

If . . .

$\triangle ABC$ is a right triangle



Then . . .

$(\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hypotenuse})^2$

$a^2 + b^2 = c^2$

You will prove Theorem 8-1 in Exercise 49.

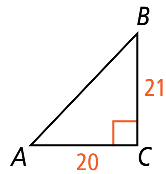
A **Pythagorean triple** is a set of nonzero whole numbers a, b, and c that satisfy the equation $a^2 + b^2 = c^2$. Below are some common Pythagorean triples.

3, 4, 5   5, 12, 13   8, 15, 17   7, 24, 25

If you multiply each number in a Pythagorean triple by the same whole number, the three numbers that result also form a Pythagorean triple. For example, the Pythagorean triples 6, 8, 10, and 9, 12, 15 each result from multiplying the numbers in the triple 3, 4, 5 by a whole number.

Problem 1 Finding the Length of the Hypotenuse

What is the length of the hypotenuse of $\triangle ABC$? Do the side lengths of $\triangle ABC$ form a Pythagorean triple? Explain.



$(\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hypotenuse})^2$ Pythagorean Theorem

$a^2 + b^2 = c^2$

$21^2 + 20^2 = c^2$ Substitute 21 for a and 20 for b.

$441 + 400 = c^2$ Simplify.

$841 = c^2$

$c = 29$ Take the positive square root.

Think

Is the answer reasonable?
Yes. The hypotenuse is the longest side of a right triangle. The value for c, 29, is greater than 20 and 21.

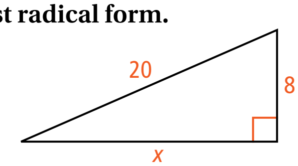
The length of the hypotenuse is 29. The side lengths 20, 21, and 29 form a Pythagorean triple because they are whole numbers that satisfy $a^2 + b^2 = c^2$.

Got It? 1. a. The legs of a right triangle have lengths 10 and 24. What is the length of the hypotenuse?

b. Do the side lengths in part (a) form a Pythagorean triple? Explain.

Problem 2 Finding the Length of a Leg

Algebra **What is the value of x? Express your answer in simplest radical form.**



$a^2 + b^2 = c^2$ Pythagorean Theorem

$8^2 + x^2 = 20^2$ Substitute.

$64 + x^2 = 400$ Simplify.

$x^2 = 336$ Subtract 64 from each side.

$x = \sqrt{336}$ Take the positive square root.

$x = \sqrt{16(21)}$ Factor out a perfect square.

$x = 4\sqrt{21}$ Simplify.

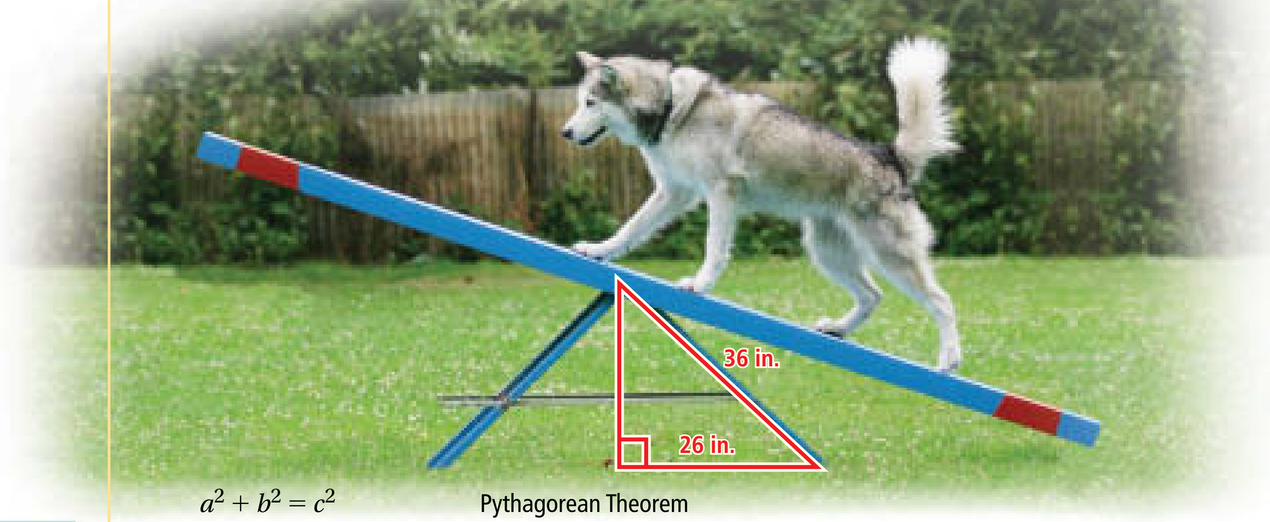
Plan

Which side lengths do you have?
Remember from Chapter 4 that the side opposite the 90° angle is always the hypotenuse. So you have the lengths of the hypotenuse and one leg.

Got It? 2. The hypotenuse of a right triangle has length 12. One leg has length 6. What is the length of the other leg? Express your answer in simplest radical form.

Problem 3 Finding Distance

Dog Agility **Dog agility courses often contain a seesaw obstacle, as shown below. To the nearest inch, how far above the ground are the dog's paws when the seesaw is parallel to the ground?**



$a^2 + b^2 = c^2$	Pythagorean Theorem
$26^2 + b^2 = 36^2$	Substitute.
$676 + b^2 = 1296$	Simplify.
$b^2 = 620$	Subtract 676 from each side.
$b \approx 24.8997992$	Use a calculator to take the positive square root.

The dog's paws are 25 in. above the ground.

Think

How do you know when to use a calculator?
This is a real-world situation. Real-world distances are not usually expressed in radical form.

Got It? **3.** The size of a computer monitor is the length of its diagonal. You want to buy a 19-in. monitor that has a height of 11 in. What is the width of the monitor? Round to the nearest tenth of an inch.

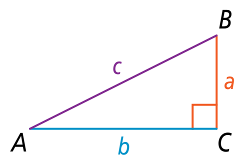
You can use the Converse of the Pythagorean Theorem to determine whether a triangle is a right triangle.

take note

Theorem 8-2 Converse of the Pythagorean Theorem

Theorem
If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

If . . .
$a^2 + b^2 = c^2$



Then . . .
$\triangle ABC$ is a right triangle

You will prove Theorem 8-2 in Exercise 52.

Plan

How do you know where each of the side lengths goes in the equation?
Work backward. If the triangle is a right triangle, then the hypotenuse is the longest side. So use the greatest number for c.

Problem 4 Identifying a Right Triangle

A triangle has side lengths 85, 84, and 13. Is the triangle a right triangle? Explain.

$a^2 + b^2 \stackrel{?}{=} c^2$	Pythagorean Theorem
$13^2 + 84^2 \stackrel{?}{=} 85^2$	Substitute 13 for a, 84 for b, and 85 for c.
$169 + 7056 \stackrel{?}{=} 7225$	Simplify.
$7225 = 7225$ ✓	

Yes, the triangle is a right triangle because $13^2 + 84^2 = 85^2$.

Got It? **4. a.** A triangle has side lengths 16, 48, and 50. Is the triangle a right triangle? Explain.

b. Reasoning Once you know which length represents the hypotenuse, does it matter which length you substitute for a and which length you substitute for b? Explain.

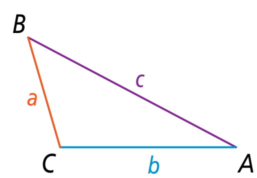
The theorems below allow you to determine whether a triangle is acute or obtuse. These theorems relate to the Hinge Theorem, which states that the longer side is opposite the larger angle and the shorter side is opposite the smaller angle.

take note

Theorem 8-3

Theorem
If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.

If . . .
$c^2 > a^2 + b^2$



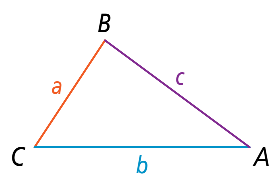
Then . . .
$\triangle ABC$ is obtuse

You will prove Theorem 8-3 in Exercise 53.

Theorem 8-4

Theorem
If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute.

If . . .
$c^2 < a^2 + b^2$



Then . . .
$\triangle ABC$ is acute

You will prove Theorem 8-4 in Exercise 54.

Problem 5 Classifying a Triangle

Plan

What information do you need?
You need to know how the square of the longest side compares to the sum of the squares of the other two sides.

A triangle has side lengths 6, 11, and 14. Is it *acute*, *obtuse*, or *right*?

$c^2 \;\blacksquare\; a^2 + b^2$	Compare c^2 to $a^2 + b^2$.
$14^2 \;\blacksquare\; 6^2 + 11^2$	Substitute the greatest value for c.
$196 \;\blacksquare\; 36 + 121$	Simplify.
$196 > 157$	

Since $c^2 > a^2 + b^2$, the triangle is obtuse.

Got It? 5. Is a triangle with side lengths 7, 8, and 9 *acute*, *obtuse*, or *right*?

Lesson Check

Do you know HOW?

What is the value of x in simplest radical form?

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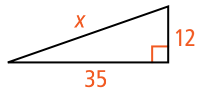

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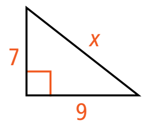

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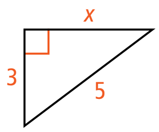

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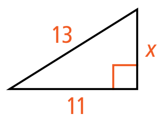


Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Vocabulary Describe the conditions that a set of three numbers must meet in order to form a Pythagorean triple.

6. Error Analysis A triangle has side lengths 16, 34, and 30. Your friend says it is not a right triangle. Look at your friend's work and describe the error.

$16^2 + 34^2 \stackrel{?}{=} 30^2$

$256 + 1156 \stackrel{?}{=} 900$

$1412 \neq 900$

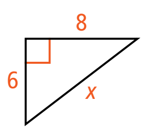
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Algebra Find the value of x.

See Problem 1.

7.



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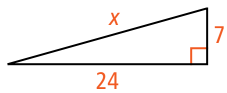

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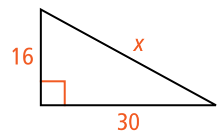

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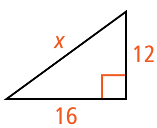

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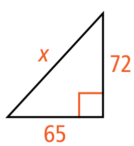

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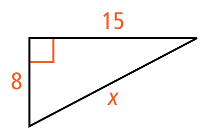


Does each set of numbers form a Pythagorean triple? Explain.

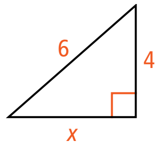
13. 4, 5, 6

14. 10, 24, 26

15. 15, 20, 25

Algebra **Find the value of x. Express your answer in simplest radical form.** See Problem 2.

16.



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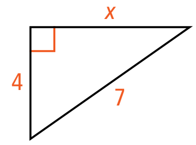

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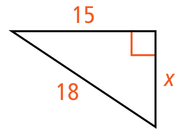

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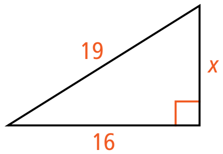

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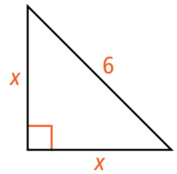

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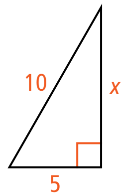
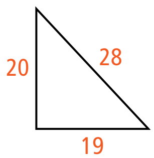


22. **Home Maintenance** A painter leans a 15-ft ladder against a house. The base of the ladder is 5 ft from the house. To the nearest tenth of a foot, how high on the house does the ladder reach? See Problem 3.

23. A walkway forms one diagonal of a square playground. The walkway is 24 m long. To the nearest meter, how long is a side of the playground?

Is each triangle a right triangle? Explain. See Problem 4.

24.



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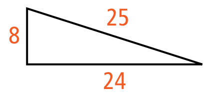

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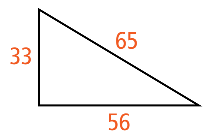


The lengths of the sides of a triangle are given. Classify each triangle as *acute*, *right*, or *obtuse*. See Problem 5.

27. 4, 5, 6

28. 0.3, 0.4, 0.6

29. 11, 12, 15

30. $\sqrt{3}$, 2, 3

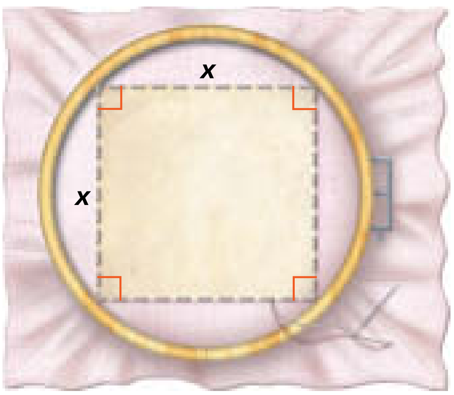
31. 30, 40, 50

32. $\sqrt{11}$, $\sqrt{7}$, 4

B Apply

33. **Think About a Plan** You want to embroider a square design. You have an embroidery hoop with a 6-in. diameter. Find the largest value of x so that the entire square will fit in the hoop. Round to the nearest tenth.

- What does the diameter of the circle represent in the square?
- What do you know about the sides of a square?
- How do the side lengths of the square relate to the length of the diameter?



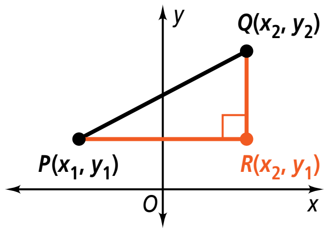
34. In parallelogram $RSTW$, $RS = 7$, $ST = 24$, and $RT = 25$. Is $RSTW$ a rectangle? Explain.

35. **Coordinate Geometry** You can use the Pythagorean Theorem to prove the Distance Formula. Let points $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the endpoints of the hypotenuse of a right triangle.
Proof

a. Write an algebraic expression to complete each of the following: $PR = \underline{\ ?\ }$ and $QR = \underline{\ ?\ }$.

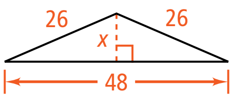
b. By the Pythagorean Theorem, $PQ^2 = PR^2 + QR^2$. Rewrite this statement by substituting the algebraic expressions you found for PR and QR in part (a).

c. Complete the proof by taking the square root of each side of the equation that you wrote in part (b).



Algebra **Find the value of x. If your answer is not an integer, express it in simplest radical form.**

36.



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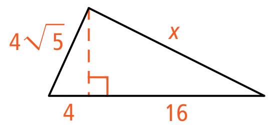

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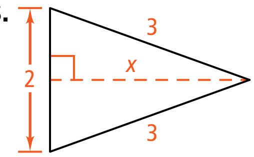


For each pair of numbers, find a third whole number such that the three numbers form a Pythagorean triple.

39. 20, 21 **40.** 14, 48 **41.** 13, 85 **42.** 12, 37

Open-Ended **Find integers j and k such that (a) the two given integers and j represent the side lengths of an acute triangle and (b) the two given integers and k represent the side lengths of an obtuse triangle.**

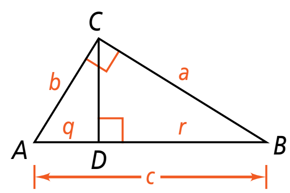
43. 4, 5 **44.** 2, 4 **45.** 6, 9 **46.** 5, 10 **47.** 6, 7 **48.** 9, 12

49. Prove the Pythagorean Theorem.
Proof

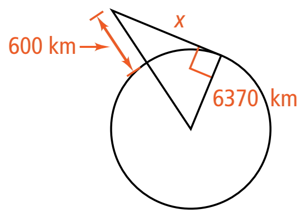
Given: $\triangle ABC$ is a right triangle.

Prove: $a^2 + b^2 = c^2$

(*Hint:* Begin with proportions suggested by Theorem 7-3 or its corollaries.)



STEM **50.** **Astronomy** The Hubble Space Telescope orbits 600 km above Earth's surface. Earth's radius is about 6370 km. Use the Pythagorean Theorem to find the distance x from the telescope to Earth's horizon. Round your answer to the nearest ten kilometers. (Diagram is not to scale.)



51. Prove that if the slopes of two lines have product -1, then the lines are perpendicular. Use parts (a)–(c) to write a coordinate proof.

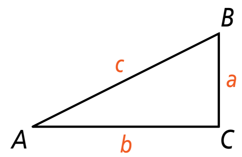
a. First, argue that neither line can be horizontal or vertical.

b. Then, tell why the lines must intersect. (*Hint:* Use indirect reasoning.)

c. Place the lines in the coordinate plane. Choose a point on ℓ_1 and find a related point on ℓ_2. Complete the proof.

Challenge

52. Use the plan and write a proof of Theorem 8-2 (Converse of the Pythagorean Theorem).
Proof

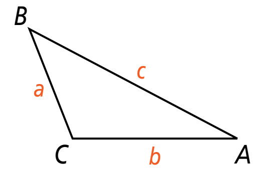


Given: $\triangle ABC$ with sides of length a, b, and c, where $a^2 + b^2 = c^2$

Prove: $\triangle ABC$ is a right triangle.

Plan: Draw a right triangle (not $\triangle ABC$) with legs of lengths a and b. Label the hypotenuse x. By the Pythagorean Theorem, $a^2 + b^2 = x^2$. Use substitution to compare the lengths of the sides of your triangle and $\triangle ABC$. Then prove the triangles congruent.

53. Use the plan and write a proof of Theorem 8-3.
Proof

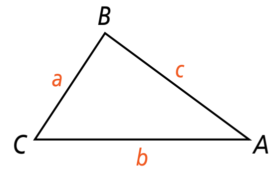


Given: $\triangle ABC$ with sides of length a, b, and c, where $c^2 > a^2 + b^2$

Prove: $\triangle ABC$ is an obtuse triangle.

Plan: Draw a right triangle (not $\triangle ABC$) with legs of lengths a and b. Label the hypotenuse x. By the Pythagorean Theorem, $a^2 + b^2 = x^2$. Use substitution to compare lengths c and x. Then use the Converse of the Hinge Theorem to compare $\angle C$ to the right angle.

54. Prove Theorem 8-4.
Proof



Given: $\triangle ABC$ with sides of length a, b, and c, where $c^2 < a^2 + b^2$

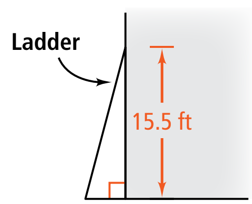
Prove: $\triangle ABC$ is an acute triangle.

Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

55. A 16-ft ladder leans against a building, as shown. To the nearest foot, how far is the base of the ladder from the building?



56. What is the measure of the complement of a 67° angle?

57. The measure of the vertex angle of an isosceles triangle is 58. What is the measure of one of the base angles?

58. The length of rectangle $ABCD$ is 4 in. The length of similar rectangle $DEFG$ is 6 in. How many times greater than the area of $ABCD$ is the area of $DEFG$?

Mixed Review

59. $\triangle ABC$ has side lengths $AB = 8$, $BC = 9$, and $AC = 10$. Find the lengths of the segments formed on $\overline{BC}$ by the bisector of $\angle A$.

See Lesson 7-5.

Get Ready! **To prepare for Lesson 8-2, do Exercises 60–62.**

Simplify each expression.

See Review, p. 399.

60. $\sqrt{9} \div \sqrt{3}$

61. $30 \div \sqrt{2}$

62. $\frac{16}{\sqrt{3}}$